Some Remarks on Solvable Lie Superalgebras

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Super Lie Algebras, Solvable Lie Algebras

Key Words

16330, 16555

AMS Classification

Superalgebras having two dimensional odd component. A classification is given of all solvable Lie superalgebras. New invariants are associated to (solvable) Lie Superalgebras.

Abstract

Some Remarks on
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Introduction

For higher dimensional $A$, the similar classification results for solvable not completely solvable Lie algebras. We hope that the methods and results of this paper will be useful in other.

We define a representation of the Lie algebra $A$ by the ideal $I$.

In the final section we apply the obtained methods to classify all Lie algebras.

In section 2 we study how the superalgebra descends if we drop the conditions that the superalgebra be a Lie algebra.

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Deformations of the Superstructures
**2.1 Artin's Theorem for the Superalgebras**

Study the theory of the Frobenius map and the projection map. The Frobenius action on the superalgebra is given by the action of the Frobenius. Clearly, the Frobenius action is the identity of the choice of basis. In this odd case, the Frobenius action is given by the identity of the choice of basis. For all \( q \in \mathbb{Q} \), the Frobenius action is given by the identity of the choice of basis. For all \( q \in \mathbb{Q} \), the Frobenius action is given by the identity of the choice of basis. For all \( q \in \mathbb{Q} \), the Frobenius action is given by the identity of the choice of basis.

We want to investigate the variety \( \mathcal{S}^{\text{rep}} \) and the projective superspace consisting of the superalgebras \( (q', \Lambda) \) which is the closed subvariety...
where $G$ is the trivial (augmentation) representation and $A$ is determined via

$$0 \rightarrow \mathbb{C} \rightarrow \Lambda \rightarrow \mathbb{C} \rightarrow 0$$

and $\mathcal{N}$ is the direct sum of $\mathcal{J}$ and $\mathcal{J}'$. Consider the non-trivial extension $\Lambda = \mathcal{J} \oplus \mathcal{J}'$. 

**Example** Let $\Lambda$ be the one-dimensional non-Abelian Lie algebra $\mathfrak{sl}(2)$. 

Substitute the example below. 

(The Lie algebra $\mathfrak{sl}(2)$ is not always true, since $\mathfrak{sl}(2)$ determines a new structure.$\square$)

Observe that we do not get a full substitute for this result. This is:

- Conclude the proof:

$$\begin{pmatrix}
\mathcal{J} & 0 \\
0 & 0
\end{pmatrix} = 
\begin{pmatrix}
I & 0 \\
0 & 1-\beta
\end{pmatrix} \cdot 
\begin{pmatrix}
0 & 0 \\
0 & 1-\beta
\end{pmatrix}$$

$$\begin{pmatrix}
\mathcal{M} & 0 \\
\lambda & \mathcal{N}
\end{pmatrix} = 
\begin{pmatrix}
I & 0 \\
0 & 1-\beta
\end{pmatrix} \cdot 
\begin{pmatrix}
\mathcal{M} & 0 \\
\lambda & \mathcal{N}
\end{pmatrix}$$

- Consider the following point in the orbit:

$$\begin{pmatrix}
\mathcal{J} & \mathcal{A} \\
\mathcal{A} & 0
\end{pmatrix} = 
\begin{pmatrix}
2 & 0 \\
0 & 1-\beta
\end{pmatrix} \cdot 
\begin{pmatrix}
\mathcal{J} & \mathcal{A} \\
\mathcal{A} & 0
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\begin{pmatrix}
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0 & 1-\beta
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\begin{pmatrix}
\mathcal{J} & \mathcal{A} \\
\mathcal{A} & 0
\end{pmatrix} \cdot 
\begin{pmatrix}
2 & 0 \\
0 & 1-\beta
\end{pmatrix}$$

**Proof:** Let $\Lambda$ be the action of $\mathfrak{sl}(2)$ on $\mathcal{M}$ be determined by the matrix

$$\begin{pmatrix}
\mathcal{M} \\
\lambda & \mathcal{N}
\end{pmatrix}$$
Then \((V, b)\) is a Lie superalgebra over \(b\) if
\[
M_b = \begin{pmatrix} 0 & -\frac{x}{z} \\ -\frac{x}{z} & 0 \end{pmatrix}
\]

Whereas \((\mathbb{C} e_1, 0)\) is obviously a Lie superalgebra over \(b\), this is not the case for \((\mathbb{C} e_2, x)\) as
\[
-\frac{y}{z} = [y, b(e_2, e_2)] \neq 2b(y, e_2, e_2) = 0
\]

Hence, a Lie superstructure \((V, b)\) over \(g\) does not necessarily induce a Lie superstructure on the semi-simplification of \(V\). Still, theorem 1 allows one to fill the matrix \(M_b\) from top left till bottom right provided we have a good flag of \(g\)-subrepresentations on \(V\).

2.2 \(V\) a semi-simple \(g\)-representation

In this section we want to describe the fiber of the projection morphism
\[
p : \text{Sup}_m(g) \rightarrow \text{Rep}_m(g)
\]
over a point of \(\text{Rep}_m(g)\) corresponding to a semi-simple \(m\)-dimensional representation \(V\) having one-dimensional simple factors. Observe that when \(g\) is a solvable Lie algebra all simple representations are one-dimensional.

Hence, we assume
\[
V = \bigoplus_{i=1}^m \mathbb{C} e_i
\]
with
\[
x_j \cdot e_i = \lambda_i(x_j)e_i
\]
for all \(1 \leq i \leq m\) and \(1 \leq j \leq n\). Clearly, \(\lambda_i\) is a weight of \(g\), i.e. \(\lambda_i \in g^*\) such that \(\lambda_i | [g, g] = 0\).

Proposition 1 Let \(V\) the \(m\)-dimensional semi-simple \(g\)-representation determined by \((\lambda_1, \ldots, \lambda_m) \in \text{Rep}_1(g)^m\). Then the fiber \(p^{-1}(V)\) of the projection map
\[
p : \text{Sup}_m(g) \rightarrow \text{Rep}_m(g)
\]
consists of all \((V, b)\) with
\[
b(e_i, e_j) \in g^\lambda_i + \lambda_j \cap g^\lambda_i \cap \text{Ann}_g(V)
\]
In particular, the fiber is a vectorspace of dimension
\[
\text{dim}(p^{-1}(V)) = \sum_{i \leq j} \text{dim}(g^\lambda_i + \lambda_j \cap g^\lambda_i \cap \text{Ann}_g(V))
\]
Then we have:

\[
(f(x), g(y))^T \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} p \end{bmatrix}
\]

The vector space of all matrices $A$ above for each $p \in \mathbb{R}$ is the following:

**Proposition 2:** With notations as above, for each $p \in \mathbb{R}$ the following holds:

However, we do not have the following result which in practice limits the possibilities. This time the information we can obtain about the matrix elements $p_{i,j}$, $i \leq j$, with $\xi \in \mathbb{R}$ forming the representation $\lambda$ is:

\[
\sum_{i=1}^{m} \sum_{j=1}^{m} \xi_{i,j} = \alpha \cdot \xi
\]

In each $\xi \in \mathbb{R}$ such that for all $i \leq j \in \mathbb{N}$ and all $\xi_{i,j}$, the equation $\sum_{i=1}^{m} \sum_{j=1}^{m} \xi_{i,j} \lambda = \lambda \xi$ is triangular, that is we have $\lambda \in \text{End}_\mathbb{R}(V)$ and the action of $\lambda$ on $V$ is triangular. That is we have $\lambda_\mathbb{R} \in \text{End}_\mathbb{R}(V)$.

Hence, we may assume that $\xi = 0$ above the diagonal of the linear morphism $\lambda$.

In this section we investigate the above the representation $\lambda$ of the projection morphism

2.3. A triangular representation

\[
\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} p \end{bmatrix}
\]

**Proof:** From the $\mathbb{R}$-linearity of the bilinear mapping we deduce...
3.1 The Rank Varities

The rank varieties of $Gr(k, n)$ are maximal order having finite global dimension.

We will show however that the global dimension of $Gr(k, n)$ is finite if and only if $k$ is prime.

Let $k > 2$ and even, then it is prime if and only if $k - 1$ is a prime.

For example, $k = 2$. Moreover, there are some notable differences. For example, $k = 3$.

However, there are some notable differences.

From work of J. Dieudonné and K. Wehrfritz, we know that $k > 2$.

\[
(f^r x^s)q - f^r x^s \leq f^r x^s + f^r x^s \leq f^r x^s
\]

\[
(f^r x^s)q - f^r x^s \leq f^r x^s - f^r x^s \leq f^r x^s
\]
\[ P = \{ x \mid x \in \mathbb{R} \} \]

**Example 2** Let \( R \) be the 5-dimensional nilpotent Lie algebra with nonzero brackets. To the superalgebra \( S \) one can associate the invariants

\[ \Gamma(x, y, z) = \begin{pmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{pmatrix} \]

The second part states that in must satisfy the inequality

\[ \dim \mathfrak{g}_\mathfrak{g} \leq \dim \mathfrak{g}_\mathfrak{f} + \dim \mathfrak{g}_\mathfrak{h} \]

where \( \mathfrak{g}_\mathfrak{g} \) is the nilpotent Lie algebra generated by a Lie ideal in \( \mathfrak{g} \). The entries of the matrix \( M \) generate a Lie ideal in \( \mathfrak{g} \). The determinant \( \det(M) = \det(\mathfrak{g}) \) is a \( R \)-semi-invariant of \( \mathfrak{g} \). Hence, the matrix \( M \) is an \( R \)-semi-invariant of \( \mathfrak{g} \).

**Corollary** With notations as above we have

\[ \mathfrak{g} = \mathfrak{g}_\mathfrak{h} \oplus \mathfrak{g}_\mathfrak{f} \oplus \mathfrak{g}_\mathfrak{g} \]

In particular, if we restrict to the two extreme cases (i.e., when \( \mathfrak{g}_\mathfrak{h} = 0 \), then \( \mathfrak{g} = \mathfrak{g}_\mathfrak{f} \oplus \mathfrak{g}_\mathfrak{g} \).

Proposition 3 If \( \mathfrak{g} \) is a homogeneous \( R \)-invariant ideal of \( \mathfrak{g} \) where \( \mathfrak{g} \) is the Lie algebra associated to some Lie algebra \( \mathfrak{h} \) for some Lie algebra \( \mathfrak{g} \), then \( \mathfrak{g} \) is also a homogeneous \( R \)-invariant ideal of \( \mathfrak{g} \).

For all \( i \geq 1 \), \( j \geq 1 \) and all \( a \) and \( b \) in the following

\[ \mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3 \]

In general, an immediate consequence of invariance of the bilinear form
Proof: All entries of the product matrix are linear forms in $z$. It is clear that $z = [z, 1, x]$ is a maximal order containing $a$ with global dimension $n$. Hence, $\text{dim } \mathcal{O} = n$.

Proposition 4. Let $s \in \text{det}(\mathcal{O})$ be a non-zero determinant of $\text{det}(\mathcal{O})$. Then $\text{det}(\mathcal{O})$ has non-zero determinant in $\text{det}(\mathcal{O})$. The structure of Clifford algebras is well-known (cf. [13, Ch. 2]) and hence is the Clifford algebra of a quadratic (signature) form over $\mathbb{R}$.

The proof for non-degenerate quadratic forms is not true for degenerate forms. The structure of Clifford algebras is well-known (cf. [13, Ch. 2]) and hence is the Clifford algebra of a quadratic (signature) form over $\mathbb{R}$.

The good microlocalization of $\mathcal{O}$. The good microlocalization of $\mathcal{O}$ is defined as: $\mathcal{O} \mathcal{D} \mathcal{D} \mathcal{O}$, where $\mathcal{D}$ is the sheaf of differential operators on $\mathcal{O}$. The good microlocalization is a right exact functor and is contravariant in $\mathcal{O}$. In case $\mathcal{O}$ is coherent, the good microlocalization is the right exact functor of $\mathcal{O}$. The good microlocalization is denoted by $\mathcal{O} \mathcal{D} \mathcal{D} \mathcal{O}$ when the product matrix is connected. If we consider the product matrix as a direct sum of connected components, then the good microlocalization is the direct sum of the good microlocalizations of these components.

\[
(f \otimes g)(\lambda) = f(\lambda) \cdot g(\lambda) = \text{det}(\mathcal{O}) \cdot \text{det}(\mathcal{O}) = \text{det}(\mathcal{O}) \cdot \text{det}(\mathcal{O})
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I

Theorem 2. Let $\mathcal{O}$ be the structured Ore-set associated to the multiplicative set $\mathcal{S}$.

Let $\mathcal{I}$ be the quasideterminant of this ring with respect to the partially induced by the multiplication.

We call the quasideterminant of the product $\mathcal{S} \mathcal{O}(\mathcal{S} \mathcal{O})$ the \textit{ultraquasideterminant} of $\mathcal{S} \mathcal{O}(\mathcal{S} \mathcal{O})$ where $\mathcal{S}$ is the structured Ore-set associated to the multiplicative set $\mathcal{S}$.

Let $\mathcal{I}$ be the quasideterminant of the product $\mathcal{S} \mathcal{O}(\mathcal{S} \mathcal{O})$.

For some $\beta = (n) \in \mathcal{S}$, $\beta \in \mathcal{O}$ and $\beta \in \mathcal{O}$.

Then $\mathcal{I}$ has no semi-invariants, the semi-invariants are $\mathcal{I} = \mathcal{S}$, and the product matrix $\alpha$ is

\[
\begin{pmatrix}
  0 & \frac{1}{\mathcal{S}} \\
  \frac{1}{\mathcal{S}} & 0 \\
\end{pmatrix}
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\[
\begin{pmatrix}
  0 & 0 \\
  1 & 0 \\
\end{pmatrix}
\]
Corollary 2 Let $\forall \neq 0$ be the semimatrix of the determinant of the product of set with $(\Omega)$ set annihilation by $g$ and all $\forall \in \Omega$ is an $\Omega$-set in $(\Omega)$. Denote this set generated by $g$ and all $\forall \in \Omega$ by $(g)^*$. In particular, if $g$ is a seminvariant of $(\Omega)$ we see that the multiplicate algebra $A \otimes \mathbb{F}$ is the Jordan-Hölder algebra and that the set of Jordan-Hölder values is contained in the set of $\forall \in \Omega$.

Observe that this is just (2) since $4$ is a solution to our problem.

Proposition 2 Let $S = (\alpha, \lambda)$ be a solution to the semimatrix such that its row $v \in \mathbb{F}^t$. With this notation we have $\alpha \in \mathbb{F}^t \setminus \{0\}$ and the semimatrix $\alpha$ is an $\mathbb{F}$-valued $\alpha$-by-$\lambda$ matrix.

Recall that each row of $\alpha \in \mathbb{F}^{t} \setminus \{0\}$ corresponds to the Jordan-Hölder factor of the set of Jordan-Hölder values (the row of $\alpha \in \mathbb{F}^{t} \setminus \{0\}$ corresponds to the set of Jordan-Hölder values) in a unique way.

Consider the semimatrix of $\alpha$ on $\mathbb{F}^{t} \setminus \{0\}$ and let $v \in \mathbb{F}^{t} \setminus \{0\}$ be the address of some row of $\alpha \in \mathbb{F}^{t} \setminus \{0\}$.

Proposition 3 Observe that the same holds for any one-set containing $\alpha$ by semimatrix.

Order property: As we recalled above.
Proposition 4.1. Some Reductions

4.1. Solvable versus completely solvable
\[ R \subseteq \{ (x^1, x^2) \in R(x_1) \cap R(x_2) \mid x_1 < x_2 \} \]

with

\[
\begin{pmatrix}
0 \\
\lambda
\end{pmatrix}
\]

are represented by

\[ 0 \leftarrow 2 \lambda \leftarrow \Lambda \leftarrow 1 \lambda \leftarrow 0 \]

Hölder factor of all \( R(\lambda) \) are represented by \( \lambda, \frac{1}{\lambda} \) and \( \lambda \frac{1}{\lambda} \) are represented by \( \lambda \). (Note: a Jordan-
Lemma 2: If \( E \subseteq R(x^1) \) and \( E \subseteq R(x^2) \) are represented by \( x^1, x^2 \)

Moreover, the extensions are easy to calculate explicitly:

\[
0 = \delta = \delta - \alpha \quad \text{for some } \alpha, \gamma \quad \text{or some } \gamma \\
0 = \delta = \delta - \alpha \\
0 \leftarrow 2 \lambda \leftarrow \Lambda \leftarrow 1 \lambda \leftarrow 0
\]

non trivial extension of \( R(\lambda) \)

\[ \text{Let } \Lambda \text{ be the Jordan-Hölder factors of the admissible representation of } R(\lambda). \]

\[ \text{If } \Lambda \neq \Lambda, \text{ then there exists a } \text{non trivial extension of } R(\lambda). \]

\[ \text{Hence, } \delta = \delta + \alpha + \gamma \]

\[ \text{In the first case, } \delta = \delta + \alpha. \]

\[ \text{In the second case, } \delta = \delta + \alpha. \]

\[ \text{In the third case, } \delta = \delta + \alpha. \]

\[ \text{In the last case, } \delta = \delta + \alpha. \]

\[ \text{In the fourth case, } \delta = \delta + \alpha. \]

\[ \text{In the fifth case, } \delta = \delta + \alpha. \]

\[ \text{In the sixth case, } \delta = \delta + \alpha. \]

\[ \text{In the seventh case, } \delta = \delta + \alpha. \]

\[ \text{In the eighth case, } \delta = \delta + \alpha. \]

\[ \text{Lemmas 1 and 2: } R(\lambda) \supseteq R(\lambda). \text{ Let } \Lambda \subseteq R(\lambda) \text{ be a } 2\text{-dimensional } \]

\[ \text{Then, in a representation with Jordan-Hölder factors } \alpha, \lambda, \gamma \text{, we }
\]

\[ \text{stabilize over an algebraic subfield the algebra. In this context, we will give a case-}
\]

\[ \text{Applying the results of the Jordan sections, one could calculate all } \]

\[ \text{as completely soluble, just start the chain of ideals with } \]
where non semi-simple \( \Lambda \) can only occur in the dotted plane.

The only points \((a, b, c) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}\) where \( \Lambda \) is not necessarily semi-simple are the intersection

1. the horizontal line: \( b = \frac{a}{2} \)

2. the diagonal line: \( a + c = \frac{a}{2} \)

3. the diagonal line: \( a = b \)

Here:

- contains a non-trivial substructure, then \((a, b) \in \mathbb{R} \times \mathbb{R}\) lies on one of the following

\[ (0, 0) \mapsto \begin{cases} \text{Proposition 7:} & \text{for } \Lambda = \mathbb{R} \times \mathbb{R} \times \mathbb{R}, \text{ such that } \mathbb{R}^3 \neq 0. \text{ Let } \Lambda \text{ be a } \mathbb{R}_{\text{non-red}} \text{-representation.} \\ & \text{Combining this fact with Lemma 1 above, we obtain the following dete-} \end{cases} \]

- \( (0, 0) \mapsto \begin{cases} \text{Proposition 7:} & \text{for } \Lambda = \mathbb{R} \times \mathbb{R} \times \mathbb{R}, \text{ such that } \mathbb{R}^3 \neq 0. \text{ Let } \Lambda \text{ be a } \mathbb{R}_{\text{non-red}} \text{-representation.} & \end{cases} \]
algebra \( A \) and over \( \mathbb{C} \)2.

Algebra \( A \) is of dimension \( \geq 3 \). Over the \( 2 \)-dimensional non-\( A \)-algebra

So we limit the classification to describing all the superalgebras over the

case if \( \dim = 2 \). Otherwise, \( \dim = 3 \). If this algebra is clear that this can only happen if \( \dim \) is \( A \)-algebra.

From the last line of \( \lambda = 2 \), \( x = (2A)(x) \lambda = \mathbb{C} \). Hence, the symmetrization of \( x \).

where \( x, y \) and \( z \) are a basis for \( x \). Hence, the symmetrization of \( x \).

\[
\begin{pmatrix}
  x & y & z
\end{pmatrix} = \mathbb{C}
\]

Proof: Obviously \( \dim (\mathbb{C}) \leq 2 \) then we may assume that

\[
\mathbb{C}, \text{ the two-dimensional non-}\mathbb{A}\text{algebra}
\]

If \( A \) of dimension \( \geq 3 \).

of the following Lie algebras

Let \( \lambda \) be the Lie ideal of \( \mathbb{C} \) generated by the entries of \( \mathbb{A} \). When \( \lambda \) must be one

\[
(\Lambda) = (\Lambda)(q, \lambda)
\]

Let \( s \) be a Lie superalgebra over \( \mathbb{C} \) with \( \dim (\mathbb{C}) \).

Proposition 8. Let \( s \) be a Lie superalgebra over \( \mathbb{C} \). Let \( s \) be a Lie superalgebra over \( \mathbb{C} \).

We only have to consider a few cases

precisely which can occur. We only have to consider a few cases

as an application of our general results we now want to give a rather concise

as an application of our general results we now want to give a rather concise

4.2 The classification

4.2 The classification
I. Dimensional theory and for a& the here is 2-dimensional. There are no Lie superalgebras outside $\mathfrak{f}$. For each of these points has $\lambda = 0 = (\delta)\gamma$ for $\lambda \in \mathfrak{m}$ and $\delta \in \mathfrak{n}$. The condition $\lambda \in \mathfrak{m}$ entails that for an $\lambda \in \mathfrak{m}$ and $\delta \in \mathfrak{n}$, the Lie algebra $\mathfrak{g} = (\delta)\lambda$.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\gamma$</th>
<th>$\lambda$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 = (\delta)\gamma$</td>
<td>$\begin{pmatrix} 0 &amp; \delta \ \delta &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} \gamma &amp; 0 \ 0 &amp; 0 \end{pmatrix}$</td>
<td>Lie</td>
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<tr>
<td>$0 = (\delta)\gamma$</td>
<td>$\begin{pmatrix} 0 &amp; \delta \ 0 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} \gamma &amp; 0 \ 0 &amp; 0 \end{pmatrix}$</td>
<td>Type</td>
</tr>
</tbody>
</table>

The Lie superalgebra with semisimple $\lambda$ (not in $\mathfrak{f}$) are easy to determine.

We take $\gamma = 0 \neq 0$. We get the following picture.

![Diagram](image_url)

Let $\lambda$ be the Abelian Lie algebra of dimension $n$. As $0 = \pi \in \mathfrak{m}$ is the only

4.2.1. The Abelian cases
and hence one gets every degree 2 form as the determinant of $\mathcal{W}$, for a degree 2 sheaf $\mathcal{W}$ of the line bundles $\mathcal{W}_N, \mathcal{W}_N^{-1}$ and $\mathcal{W}$ to be symmetric $2 \times 2$ matrices over $\mathcal{O}_T$. For $n = 0$ and $\mathbb{Q} = \mathbb{R}$, every such bundle with $\mathcal{O}_T$ and Chern numbers $0 = 0$ and $\omega = 0$ [11] and for the smooth form, there corresponds a vector bundle over $\mathbb{R}$. \[ \text{for } (x, y) \in \mathbb{C} \quad \text{with variables } z, w \quad \text{for } (x^2 + y^2)^{\frac{1}{2}} \quad \text{and } x^2 + y^2 \quad \text{with } \quad \mathcal{O}_T \quad \text{and Chern numbers } 0 = 0 \quad \text{and } \omega = 0 \]  

For $\mathbb{A}$ and $\mathbb{Q}$ it is trivial that every quadratic form can arise as the determinate of a symmetric matrix of degree 2. Therefore, the superalgebra $\mathcal{W}$ is the trivial representation when an $\mathbb{A}$-dimensional with $\mathcal{O}_T = \mathbb{A}$ and for a $\mathbb{Q}$-dimensional one. Finally, and for $\mathbb{A} = \mathbb{Q}$ there is no such structure. For $\mathbb{A}$-dimensional with $\mathcal{O}_T = \mathbb{A}$ from the Jacob superalgebra one deduces that $\mathcal{W}$ is not the trivial representation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Cond.</th>
<th>$\mathcal{W}$</th>
<th>$\Lambda$</th>
<th>$\nu$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>point</td>
<td>$\nu = \langle \delta \rangle$</td>
<td>$\begin{pmatrix} 0 &amp; 0 \ 0 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} \sigma &amp; \delta \ \sigma &amp; \delta \end{pmatrix}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and hence $\mathcal{W} = \langle \delta \rangle$. Similarly, $\mathcal{W} = \langle \delta \rangle$. Therefore, $\mathcal{W} = \mathcal{W}$. With $\mathcal{W} \neq 0$ then $0 \neq a = (x)$. It must be important.

Recalling the study of the plane in $\mathbb{P}^2$, $\Lambda$ is not the trivial representation.
4.2.2 The Borel case

If $\mathfrak{h}$ is the 2-dimensional non-Abelian Lie algebra $[\mathfrak{h}, \mathfrak{h}] = \mathfrak{g} = 0$, then $\alpha^* \in \mathfrak{h}^*$ is the only weight such that $\mathfrak{g}^\alpha \neq 0$. Therefore, we get the following picture.

Again, the cases where $\gamma$ is semi-simple (and not one of the points $P_i$) are easy to work out.
Which gives us the following list of possible structures in the points \( P \):

\[
0 = \psi f + (\epsilon p + I - \lambda)g = \epsilon f \\
0 = \epsilon(I - \psi f) \\
0 = \psi \epsilon f + \epsilon f = \epsilon f
\]

Giving the additional restrictions:

\[
0 = \psi \epsilon f, (\epsilon p + I - \lambda)g = \epsilon f \\
0 = \epsilon \psi(\epsilon p + I - \lambda)g + \epsilon (\epsilon p + I - \lambda)g = \epsilon f \\
0 = \psi \epsilon f
\]

On top, we have to satisfy the Jacob's conditions.

\[
\frac{\partial f}{\partial g} + p \psi = f \psi + \epsilon f = 0 \quad \text{and} \quad 0 = p \epsilon + f \psi = 0
\]

\[
\frac{\partial f}{\partial g} = \left\{ \begin{array}{l} \frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \end{array} \right.
\]

\[
\frac{\partial f}{\partial g} + \frac{\partial f}{\partial g} = \left\{ \begin{array}{l} \frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\end{array} \right.
\]

$0 = p \psi + p(\epsilon p + I - \lambda)g = \epsilon f + \epsilon f = 0$ and $0 = p \epsilon + f \psi = 0$.

Giving the conditions of the scheme.

\[
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f
\]

Have the relations.

\[
\psi f + \epsilon \psi = (\epsilon p + I - \lambda)g
\]

And if there is a solution, then we have the relations.

\[
\frac{\partial f}{\partial g} = \epsilon f \\
\frac{\partial f}{\partial g} = \epsilon f
\]

As we have seen, we can take the scheme.

\[
\begin{pmatrix}
0 & 0 \\
0 & \epsilon f
\end{pmatrix}
\]

By the lemma, we know that $\epsilon f = 0$.

\[
\begin{pmatrix}
0 & \epsilon f \\
0 & \epsilon f
\end{pmatrix}
\]

Next, let us consider the non-singular case in the points $P$, i.e., $X$.

\[
\begin{array}{|c|c|c|c|c|}
\hline
X & \Lambda & \text{type} \\
\hline
0 & \epsilon f & \text{I} \\
\hline
\end{array}
\]
Consider first the non-trivial 2-dimensional $\mathbb{R}$-representation \( z = (\Lambda) \) of the adjoint representation of \( \mathfrak{g} \).

**Proposition 9.** The only non-trivial irreducible representations over \( \mathbb{R} \) are those coming from the adjoint representation of \( \mathfrak{g} \).

**Remark.** Only in the point \( P \) are there the representations \( \psi' \Lambda \) which are not completely solvable anymore when \( \psi' \in \mathfrak{g} \) and \( \Lambda \neq 0 \).

| \( \begin{pmatrix} \lambda \delta & 0 \\ 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 \\ \frac{\lambda}{\lambda} & 0 \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} \lambda \delta & 0 \\ 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 \\ \frac{\lambda}{\lambda} & 0 \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} \lambda \delta + \frac{\lambda}{\lambda} \xi \iota & - \lambda \delta \xi \iota \\ \lambda \delta \xi \iota & \lambda \delta \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 \\ - \lambda \delta \xi \iota + \lambda \delta \xi \iota \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} 0 & \lambda \delta \xi \iota \\ \lambda \delta \xi \iota & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 \\ \frac{\lambda}{\lambda} & 0 \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} \lambda \delta \xi \iota & \lambda \delta \xi \iota \\ \lambda \delta \xi \iota & \lambda \delta \end{pmatrix} \) | \( \begin{pmatrix} \lambda \delta \xi \iota & 0 \\ 0 & \frac{\lambda}{\lambda} \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} 0 & \lambda \delta \xi \iota \\ \lambda \delta \xi \iota & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 \\ \frac{\lambda}{\lambda} & 0 \end{pmatrix} \) | \( \psi \) | \( \psi' \) |
| \( \begin{pmatrix} \lambda \delta & \lambda \delta \xi \iota \\ \lambda \delta \xi \iota & \lambda \delta \end{pmatrix} \) | \( \begin{pmatrix} \lambda \delta \xi \iota & 0 \\ 0 & \frac{\lambda}{\lambda} \end{pmatrix} \) | \( \psi \) | \( \psi' \) |

**Proof.** Consider first the non-trivial 2-dimensional $\mathbb{R}$-representation \( z = (\Lambda) \) of the adjoint representation over \( \mathbb{R} \) that is not completely solvable anymore when \( \psi' \in \mathfrak{g} \) and \( \Lambda \neq 0 \).
References

The methods of this paper can be viewed as a first step towards this (grad).

The next step is the initial step...

Proof: Follows from (1) and the classification...